Geometry 11.6 Notes: Geometric Probability (pp 699–701)

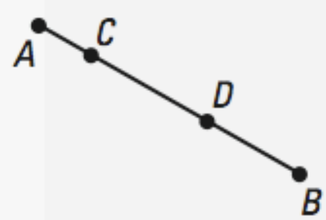
A **probability** is a number from 0 to 1 that represents the chance that an event will occur. Assuming that all outcomes are equally likely, an event with a probability of 0 *cannot* occur. An event with a probability of 1 is *certain* to occur, and an event with a probability of 0.5 is just as likely to occur as not.

In an earlier course, you may have evaluated probabilities by counting the number of favorable outcomes and dividing that number by the total number of possible outcomes. In this lesson, you will use a related process in which the division involves geometric measures such as length or area. This process is called **geometric probability**.

GEOMETRIC PROBABILITY

PROBABILITY AND LENGTH

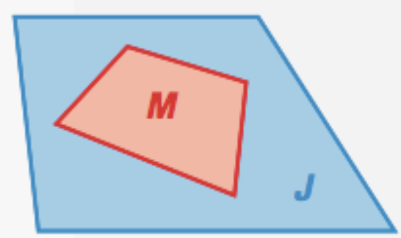
Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is as follows:



$$P(\text{Point } K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

PROBABILITY AND AREA

Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is as follows:



$$P(\text{Point } K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

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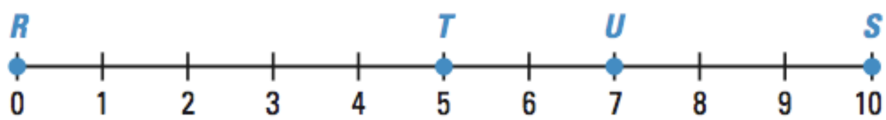
EXAMPLE 1 *Finding a Geometric Probability*

Find the probability that a point chosen at random on $\overline{RS}$ is on $\overline{TU}$.



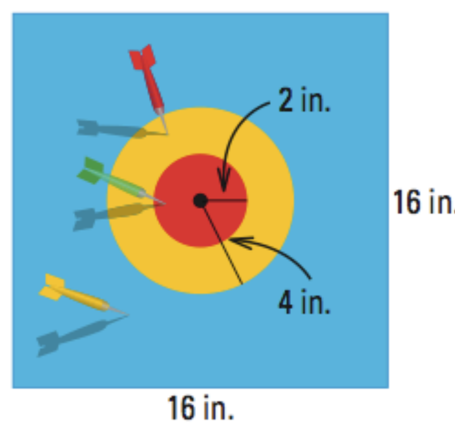
SOLUTION

$$P(\text{Point is on } \overline{TU}) = \frac{\text{Length of } \overline{TU}}{\text{Length of } \overline{RS}} = \frac{2}{10} = \frac{1}{5}$$

▶ The probability can be written as $\frac{1}{5}$, 0.2, or 20%.

EXAMPLE 2 *Using Areas to Find a Geometric Probability*

DART BOARD A dart is tossed and hits the dart board shown. The dart is equally likely to land on any point on the dart board. Find the probability that the dart lands in the red region.



SOLUTION

Find the ratio of the area of the red region to the area of the dart board.

$$\begin{aligned} P(\text{Dart lands in red region}) &= \frac{\text{Area of red region}}{\text{Area of dart board}} \\ &= \frac{\pi(2^2)}{16^2} \\ &= \frac{4\pi}{256} \\ &\approx 0.05 \end{aligned}$$

▶ The probability that the dart lands in the red region is about 0.05, or 5%.

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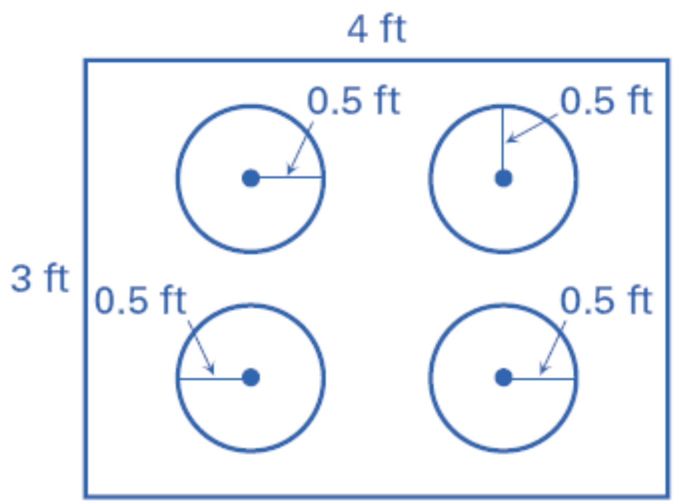
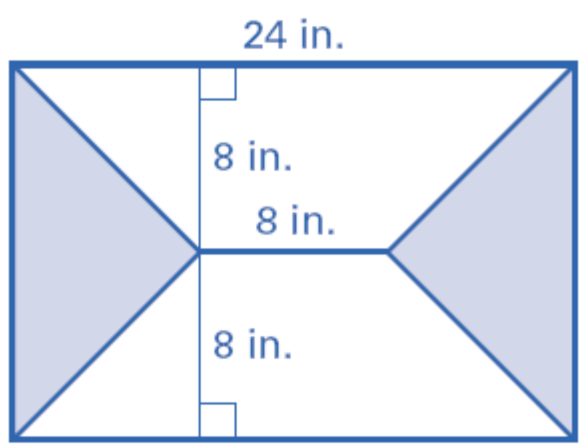
Practice.

1. Example: Each circular hole on the target has a radius of 0.5 ft. If a beanbag is equally likely to land on any point on the target, find the probability that the beanbag goes through one of the holes.



2. Find the probability that a randomly chosen point in the figure lies in the shaded region.



"You're right. Even at 7 million pixels, we all look alike."

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EXAMPLE 3 *Using a Segment to Find a Geometric Probability*

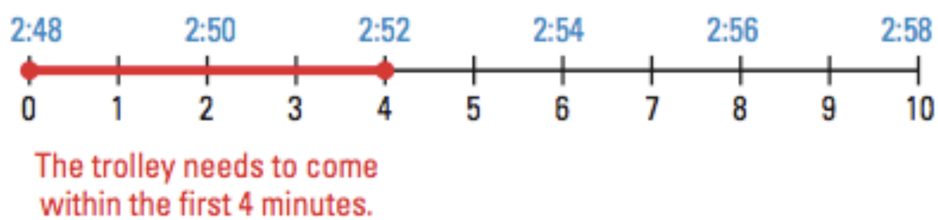
TRANSPORTATION You are visiting San Francisco and are taking a trolley ride to a store on Market Street. You are supposed to meet a friend at the store at 3:00 P.M. The trolleys run every 10 minutes and the trip to the store is 8 minutes. You arrive at the trolley stop at 2:48 P.M. What is the probability that you will arrive at the store by 3:00 P.M.?

SOLUTION

To begin, find the greatest amount of time you can afford to wait for the trolley and still get to the store by 3:00 P.M.

Because the ride takes 8 minutes, you need to catch the trolley no later than 8 minutes before 3:00 P.M., or in other words by 2:52 P.M.

So, you can afford to wait 4 minutes (2:52 − 2:48 = 4 min). You can use a line segment to model the probability that the trolley will come within 4 minutes.



$$P(\text{Get to store by 3:00}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{4}{10} = \frac{2}{5}$$

▶ The probability is $\frac{2}{5}$, or 40%.

Practice.

3. You ride to work on a bus. The buses come every 15 min and the ride to work is 31 min. You arrive at the bus stop at 8:24 A.M. What is the probability that you will be at work at 9:00 A.M.?

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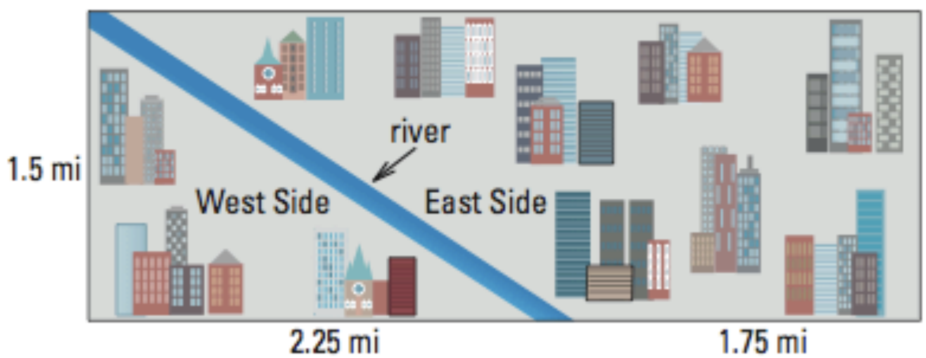
4. Buses pick up passengers at the airport every 33 min. The ride home on the bus takes 22 min. If you get to the bus stop at 7:11 P.M., what is the probability that you can get home by 8:00 P.M.?

EXAMPLE 4 *Finding a Geometric Probability*

JOB LOCATION You work for a temporary employment agency. You live on the west side of town and prefer to work there. The work assignments are spread evenly throughout the rectangular region shown. Find the probability that an assignment chosen at random for you is on the west side of town.



SOLUTION

The west side of town is approximately triangular. Its area is $\frac{1}{2} \cdot 2.25 \cdot 1.5$, or about 1.69 square miles. The area of the rectangular region is $1.5 \cdot 4$, or 6 square miles. So, the probability that the assignment is on the west side of town is

$$P(\text{Assignment is on west side}) = \frac{\text{Area of west side}}{\text{Area of rectangular region}} \approx \frac{1.69}{6} \approx 0.28.$$

▸ So, the probability that the work assignment is on the west side is about 28%.

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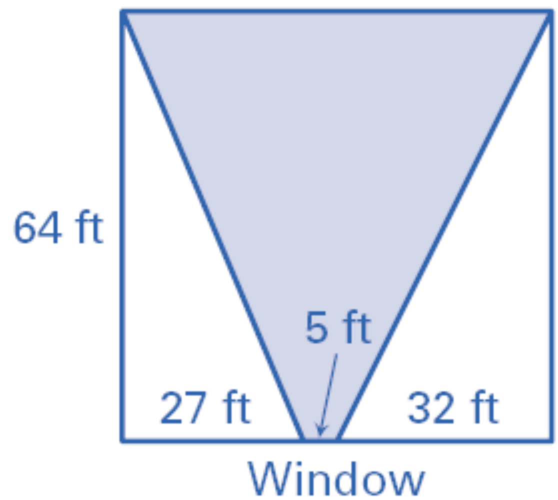
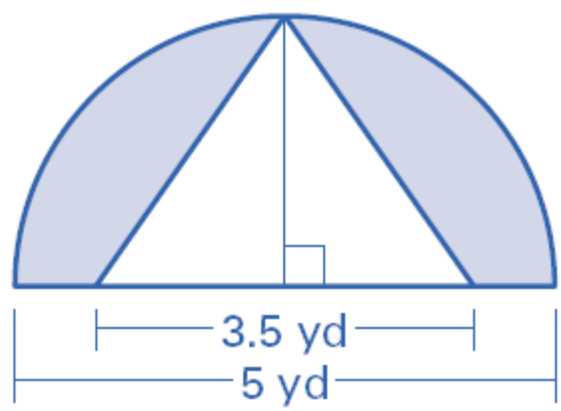
<u>Practice.</u>

5. The shaded area in the diagram of your backyard below shows the area that you can see from your window. If a child is equally likely to play in any part of the backyard, what is the probability that you will be able to see him from the window?



6. You are acting on the stage shown in the diagram below. In order to be clearly seen by the audience, you need to stay in the triangular portion of the stage. If you are equally likely to be in any part of the stage, what is the probability that you will not be clearly seen by the audience?



7. What is used to find geometric probability?